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Electric Motor Efficiency Testing Under the New Part 431 of Chapter II of Title 10, Code of Federal Regulations: Enforcement Testing

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Abstract

The provisions for electric motor efficiency testing under the proposed new Part 431 of Chapter II of Title 10, Code of Federal Regulations, as published for public comment in the *Federal Register*, Vol. 61, No. 230, Wednesday, November 27, 1996, pp. 60439–60475, are discussed. The criteria for demonstration of compliance with the energy efficiency requirements established by Energy Policy and Conservation Act of 1975, as amended, are presented. The operating characteristics, i.e., the estimated probability of demonstrating compliance based on the mean efficiency, standard deviation, and number of units tested, of the Sampling Plan for Enforcement Testing recommended by the new Part 431 are evaluated by model calculations.

Keywords

electric motor efficiency; electric motor testing; Energy Policy Act; operating characteristics; sampling plan

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Electric Motor Efficiency Testing under the Proposed Part 431 of Chapter II of Title 10, Code of Federal Regulations: Enforcement Testing

Contents

1 Introduction

This document provides commentary on the rules for efficiency testing of polyphase electric motors proposed by the new Part 431 of the Code of Federal Regulations (10 CFR Part 431) as published for public comment in the *Federal Register* [1]. The uses of actual testing, in the context of the new Part 431, are 1) for demonstration of compliance of a basic model with the levels of efficiency established by the Energy Policy and Conservation Act of 1975, as amended; 2) for substantiation of an alternative efficiency determination method (AEDM) as described in Subpart B §431.24; and 3) for enforcement testing. This document provides background information on the statistical methods recommended by the new Part 431 for enforcement testing, i.e., topic 3: Topics 1 and 2 will be discussed in a future publication.

This report supplements the materials published in the *Federal Register* and is intended for persons having an interest in motor efficiency testing and in the Energy Policy Act (EPAct) legislation, or involved in recommending motors for testing, in estimating the relative risks in choosing a specific motor for testing, in enforcement testing, or in performing efficiency measurements in support of the new Part 431. Actual testing under the proposed rule is based on the IEEE 112 [2] and CSA-390 [3] standards. The reader is referred to these documents for a discussion of the efficiency measurements.

One note of caution: This report is *not* an official statement of policy by the Department of Energy (DOE) regarding the proposed rule. This report must be regarded only as extended commentary on the recommendations contained in the proposed new Part 431. Readers are encouraged to contact the Department of Energy with questions and/or comments on the proposed new Part 431. Several points of contact are provided in Section 5 of this report.

The remainder of this document is organized by sections: Section 2 discusses the criteria for compliance of a basic model with the mandated efficiencies; Section 3 discusses the objectives and general guidelines considered in developing the Sampling Plan for Enforcement Testing provided in the new Part 431; Section 4 discusses of the operating characteristics of the Sampling Plan for Enforcement Testing; Section 5 provides points of contact for further information, and Section 6 contains a list of references. Supplemental information provided to support those discussions is contained in: Appendix A: the full text of the Sampling Plan for Enforcement Testing included in the new Part 431 [1]; Appendix B: a listing of the computer code used to model the operating characteristics of the Sampling Plan for Enforcement Testing; Appendix C: an abridged table of t-values for single-sided t-tests.

2 The criteria for demonstration of compliance by actual testing

The intent of the EPAct legislation is to improve the nation's efficient use of electric power by the use of efficient electric motors. The intent of the legislation is, therefore, satisfied if the mean full-load efficiency of the entire population of motors, of each basic model covered by the legislation equals or exceeds the statutory efficiency. Determination of the accuracy of labeled motor efficiencies, e.g., establishing upper and lower bounds for labeled motor efficiencies or estimating the true variability of the population of motors, is not strictly required to demonstrate that the EPAct objective has been met and is, therefore, beyond the purview of this legislation.

The true mean full-load efficiency is an abstract quantity that can be known only in principle since its full determination requires that the efficiencies of all units of a basic model be measured without

error. Since this is not possible, any demonstration of compliance by actual testing must rely on accepted statistical methods to estimate the confidence of achieving the mandated efficiency, i.e., to estimate the probability that the true mean full-load efficiency is not less than the mandated level. The choice of statistical confidence does not affect the quality or average performance of the motors being tested but rather indicates the method used to establish a confidence interval about the mean of the sample: The choice of confidence level is governed largely by convention. For the purpose of demonstrating compliance with the mandated efficiency levels, the Department of Energy has determined that compliance is demonstrated provided the true mean efficiency of a basic model is not less than the mandated efficiency with confidence not less than 90 percent. It should be emphasized that the confidence specified is that for the estimate of the mean efficiency and not for the efficiency of a single unit.

The criteria for demonstration of compliance provided is Subpart B §431.24 Paragraph (b)(1)(iii) of the new Part 431 follow:

> For each basic model selected for testing,[1] a sample of units shall be selected at random and tested in accordance with §§431.23 and 431.25, and appendix A, of this subpart. The sample shall be comprised of production units of the basic model, or units that are representative of such production units, and shall be of sufficient size to ensure that any represented value of the nominal or average full load efficiency of the basic model is no greater than the lesser of
>
> (A) The average full load efficiency of the sample, or
>
> (B) The lower 90 percent confidence limit of the average full load efficiency of the entire population divided by the coefficient "K" applicable to the represented value. The coefficients are set forth in appendix B of this subpart.

[1] Components of similar design may be substituted without requiring additional testing if the represented measures of energy consumption continue to satisfy the applicable sampling provision.

3 Design of the Sampling Plan for Enforcement Testing

A Sampling Plan for Enforcement Testing is provided in the proposed new Part 431 to aid in the interpretation of data obtained by actual testing and to ensure uniform application of the rule. The objectives of the Sampling Plan are: 1) to obtain an estimate of the true mean full-load efficiency, 2) to establish reasonable measurement tolerances for motor efficiencies, and 3) to ensure that results that are obtained by actual testing are significant within these tolerances.

In addition to these objectives, the Sampling Plan for Enforcement Testing should conform to the following general guidelines:

- The criteria for demonstration of compliance specified for electric motor testing should be consistent with those established for other industries and/or products covered by the EPAct legislation.
- Demonstration of compliance by actual testing should, when feasible, rely on standards and practices currently used by industry and should not require new or unique standards or procedures.
- Compliance testing should fall within the realm of normal testing for purposes of quality control, i.e., the development of unique facilities should not be required.
- The level of quality control and quality assurance required for demonstration of compliance should be supported across the industry.
- The rules for compliance testing should apply uniformly across the industry, i.e., the level of quality control and quality assurance required by the Sampling Plan should be consistent with industry practice regarding motor size and type.

What statistical test is appropriate? The legislation is supported by ensuring that the mean efficiency of each basic model is not less than the statutory full-load efficiency, SFE, i.e., the efficiency levels established by the Energy Policy and Conservation Act of 1975, as amended. This objective may be satisfied by demonstrating that the mean efficiency obtained by tests conducted on a random sample of motors exceeds a lower

bound or lower control limit. The Sampling Plan developed here seeks to estimate the true mean full-load efficiency of the basic model and the confidence that this estimate exceeds a lower control limit. The Sampling Plan must assume that the true mean full-load efficiency, the standard deviation of the motor efficiencies, and, indeed, that precise information on the distribution of motor efficiencies are not known.

The best estimate of the true mean efficiency that may be obtained by tests conducted on a random sample is the mean efficiency of that sample,

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \tag{1}$$

where X_i is the measured full-load efficiency of unit i, and n is the number of units tested. The uncertainty of this estimate depends on two factors: 1) the size of the sample, i.e., the number of motors tested, and 2) the underlying variability in the entire population of motors. The sample standard deviation,

$$S = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}, \tag{2}$$

is one measure of the variability of the motor efficiencies. The standard error in the mean,

$$SE(\bar{X}) = \frac{S}{\sqrt{n}}, \tag{3}$$

provides an estimate of the standard deviation of the mean efficiency as determined by tests conducted on samples of n units. If we assume that the efficiencies of the entire population of motors are normally distributed about the true mean full-load efficiency, μ, then the ratio

$$t = \frac{\mu - \bar{X}}{SE(\bar{X})} \tag{4}$$

is distributed according to a probability density function that is know in statistics literature as the t-distribution. The values of t associated with commonly specified percentiles are readily available and are included in many references on statistics [4], for completeness, and for the convenience of the reader, an abridged t-table is provided in Appendix C of this report.

To be mathematically precise, much of the discussion that follows is rigorously true if and only if the motor efficiencies are normally distributed. However, the t-test is well known to be insensitive to departures from the assumption of normally distributed data: The t-test is a test on a mean, i.e., an average of independent values obtained by a random sample; and, in general, sums of arbitrary, independent random values tend to have a distribution that is *almost* normal. Hence, the t-test is not strongly influenced by the exact form of the underlying distribution.

Establishing a lower control limit. The ratio t defined in eq (4) provides an expression for the mean of the sample:

$$\bar{X} = \mu - tSE(\bar{X}). \tag{5}$$

We may assume, by hypothesis, that the units to be tested are drawn from a population of motors for which the mean full-load efficiency is equal to or greater than the statutory full-load efficiency. If t is the 90th percentile of the t-distribution appropriate for the sample size, then the probability of obtaining a mean efficiency,

$$\bar{X} \geq SFE - tSE(\bar{X}), \tag{6}$$

is not less than 90 percent, which recommends the lower control limit,

$$LCL = SFE - tSE(\bar{X}). \tag{7}$$

To apply this method, a random sample of motors is tested and the mean and standard error in the mean are calculated. Based on the size of the sample and the confidence desired, the appropriate t-value is selected and the lower control limit calculated. For example, by way of reference to the t-table provided in Appendix C, 90 percent confidence and a sample size of five units yields a t-value of 1.533. Provided the mean efficiency obtained from the random sample is not less than the lower control limit, as defined by eq (7), we may assert with a confidence not less than 90 percent that the true mean efficiency of the entire population is not less than the mandated efficiency and thus that the basic model is in compliance.

In any statistical test there is some probability of incorrectly concluding noncompliance. By design, the probability that the mean efficiency for a random sample drawn from this population of

motors would fall below the lower control limit, hence, the risk of incorrectly concluding that the basic model is in noncompliance, is not greater than 10 percent.

There is some probability that the estimate of the standard deviation and, therefore, the standard error in the mean is large and that the lower control limit may be set, by chance, to a value that defeats the intent of the legislation. To avoid this circumstance, it is sufficient to establish a tolerance for the standard error in the mean, $SE(\bar{X})$. The tolerance for the standard error should be chosen to be appropriate for the size and type of motor being tested and to be supported across the industry.

The problem of defining a reasonable tolerance for the standard error. The strategy recommended here is to establish acceptable benchmarks for the lower control limit. One possible solution, is to base these tolerances on the existing NEMA guidelines for motor efficiency labeling [5]. These guidelines were developed by consensus among motor manufacturers and they are followed, on a voluntary basis, by a large segment of the motor manufacturers. Although the NEMA guidelines were developed primarily to provide uniformity in motor efficiency labeling, they are used for purposes of quality control by many manufacturers and they may therefore provide a reasonable basis for estimating efficiency tolerances among motors of different size and type. We argue that the NEMA 20 percent loss tolerance is significant and meaningful and is likely to be supported across the industry.

By definition, the efficiency as a percentage can be expressed as,

$$\mu = \frac{P_{out}}{P_{in}} 100. \tag{8}$$

The input and output power are indicated by P_{in} and P_{out}, respectively. So that the minimum efficiency associated with a 20 percent tolerance in power loss is expressed as,

$$\mu_{min} = \frac{P'_{out}}{P'_{in}} 100. \tag{9}$$

Following the convention used by NEMA [5], the minimum efficiency is calculated at constant output power, i.e., $P_{out} = P'_{out}$, thus

$$\begin{aligned} \mu_{min} &= \frac{P_{out}}{P_{in} + 0.20(P_{in} - P_{out})} 100 \\ &= \frac{\mu}{120 - 0.20\mu} 100, \end{aligned} \tag{10}$$

which is again expressed as a percentage. The lower control limit must then satisfy two conditions:

$$LCL = SFE - tSE(\bar{X}) \text{ and} \tag{11}$$

$$\geq \frac{SFE}{120 - 0.20SFE} 100. \tag{12}$$

The second condition is obtained from eq (10) by setting the efficiency equal to the SFE.

The standard deviation allowed under the Sampling Plan. An estimate of the width of the distribution of motor efficiencies, i.e., the standard deviation, that is allowed by the Sampling Plan may be obtained by equating the right-hand sides of eqs (11) and (12):

$$\frac{SFE}{120 - 0.20SFE} = SFE - tSE(\bar{X}), \tag{13}$$

which, after substitution from eq (3), may be rearranged to yield

$$S = \frac{\sqrt{n}}{t} \frac{SFE(20 - 0.20SFE)}{120 - 0.20SFE}. \tag{14}$$

It should be noted that the NEMA minimum efficiencies are determined by a table [5] while the new Part 431 relies on a formula. A comparison of the NEMA guidelines and the LCL recommended by the new Part 431 is presented in Tab. 1. The column headed "NEMA 20 percent Loss Tolerance" lists 20 percent of the difference $SFE - \mu_{min}$ while the column headed "Part 431 20 percent Loss Tolerance" lists 20 percent of the difference $SFE - LCL$. The maximum standard deviation is calculated using eq (14) by assuming a sample of five units.

Advantages of the Sampling Plan. By design, the tolerances for the motor efficiency specified by the rule are closely associated with the NEMA guidelines for motor efficiency labeling, and are thus likely to follow quality control practices used by industry. This has several potential

Table 1. Comparison of the NEMA 20 percent loss tolerance, the Part 431 20 percent loss tolerance, and the maximum standard deviation allowed under the Sampling Plan.

Statutory Full-load Efficiency	NEMA Minimum Efficiency	NEMA 20 percent Loss Tolerance	Part 431 *LCL*	Part 431 20 percent Loss Tolerance	Maximum Standard Deviation
75.5	72.0	3.5	72.0	3.5	5.1
80.0	77.0	3.0	77.0	3.1	4.5
82.5	80.0	2.5	80.1	2.8	4.1
84.0	81.5	2.5	81.4	2.6	3.8
85.5	82.5	3.0	83.1	2.4	3.5
86.5	84.0	2.5	84.2	2.3	3.4
87.5	85.5	2.0	85.4	2.1	3.1
88.5	86.5	2.0	86.5	2.0	2.9
89.5	87.5	2.0	87.7	1.8	2.6
90.2	88.5	1.7	88.4	1.8	2.6
91.0	89.5	1.5	89.4	1.6	2.2
91.7	90.2	1.5	90.2	1.5	2.2
92.4	91.0	1.4	91.0	1.5	2.2
93.0	91.7	1.3	91.7	1.3	1.9
93.6	92.4	1.2	92.4	1.2	1.8
94.1	93.0	1.1	93.0	1.1	1.6
94.5	93.6	0.9	93.5	1.0	1.5
95.0	94.1	0.9	94.1	0.9	1.3

advantages: 1) industry should be better able to estimate the risk involved with the selection of a basic model for testing and thus better manage their financial risk and 2) the investment required for personnel training should be reduced since the tolerances recommended by the new Part 431 follow those currently used by industry.

The Sampling Plan has an additional advantage: If a manufacturer is in compliance with the voluntary NEMA guidelines for motor efficiency labeling, i.e., 1) the average full-load efficiency of the entire population of motors exceeds the NEMA nominal efficiency, and 2) no single unit falls below the NEMA minimum efficiency, then the probability of demonstrating compliance by actual testing is high.

Risks associated with adopting the Sampling Plan. We have assumed that the NEMA guidelines reasonably represent the tolerance that is maintained across the industry. There are, indeed, very little data to test the validity of this assumption.

4 Operating characteristics of the Sampling Plan for Enforcement Testing

How the Sampling Plan will perform in practice depends, to a large extent, on the type and magnitudes of the variabilities encountered in the efficiency measurements. Unfortunately, there are limited data on which to base these judgments. The Sampling Plan may be modeled, however, and tested, in a sense, on worst-case or other scenarios. A computer-based computation was developed for this purpose and is included in Appendix B of this report. The program calculates the probability of demonstrating compliance based on the true mean efficiency, the sample size, and standard deviation. These calculations assume that the full-load efficiencies are normally distributed.

The results of several calculations are plotted in figures 1 and 2. The data shown are for standard deviations equal to 20 percent, 10 percent, and 7 percent of the total loss, i.e., the 20 percent loss tolerance corresponds to 1, 2, and 3 standard deviations. These figures plot the probability of demonstrating compliance as a function of the true mean efficiency. For example, by way of reference to the lower panel in figure 1, the probability of demonstrating compliance for samples of five motors drawn at random from a population having a true mean efficiency of 76 percent is approximately 0.98, where a probability of 1.0 provides complete certainty of compliance. Thus the risk to the manufacturer of a false outcome would be 2 in 100. For these same conditions, the probability that a basic model having a true mean efficiency of 74.5 percent would demonstrate compliance is approximately 0.50.

5 Further information

For information regarding the status of the proposed new Part 431 and for details regarding pub-

lic review of the new Part 431 contact:

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The FORTRAN code used for the model calculations presented here and documented in Appendix B is available on request.

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6 References

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[3] Energy Efficiency Test Methods for Three-Phase Induction Motors, C390-93, 1993, Canadian Standards Association, Rexdale, Ontario, Canada.

[4] See, for example, R.S. Schulman, *Statistics in Plain English with Computer Applications*,Van Norstand Reinhold, New York, NY, USA, 1992.

[5] Motors and Generators, MG 1-1993, 1993, National Electrical Manufacturers Association, Washington, DC, USA.

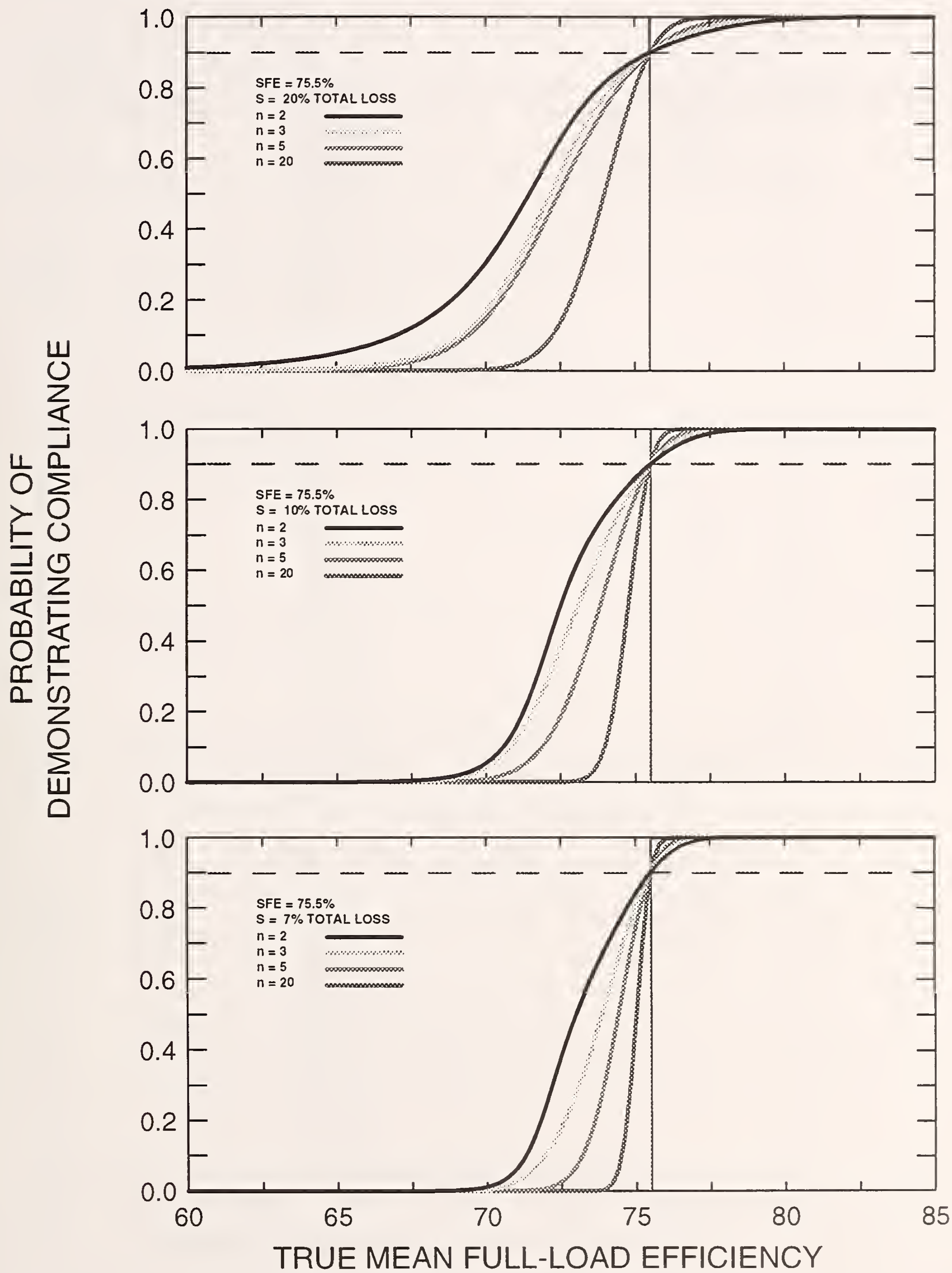

Figure 1. The probability of demonstrating compliance for sample size, n, and standard deviation , S. The statutory full-load efficiency, SFE, is 75.5 percent. The motor efficiencies are assumed to be normally distributed with the standard deviation expressed as a percentage of the total losses, i.e., $(100 - \mu)$, where μ is the true mean full-load efficiency.

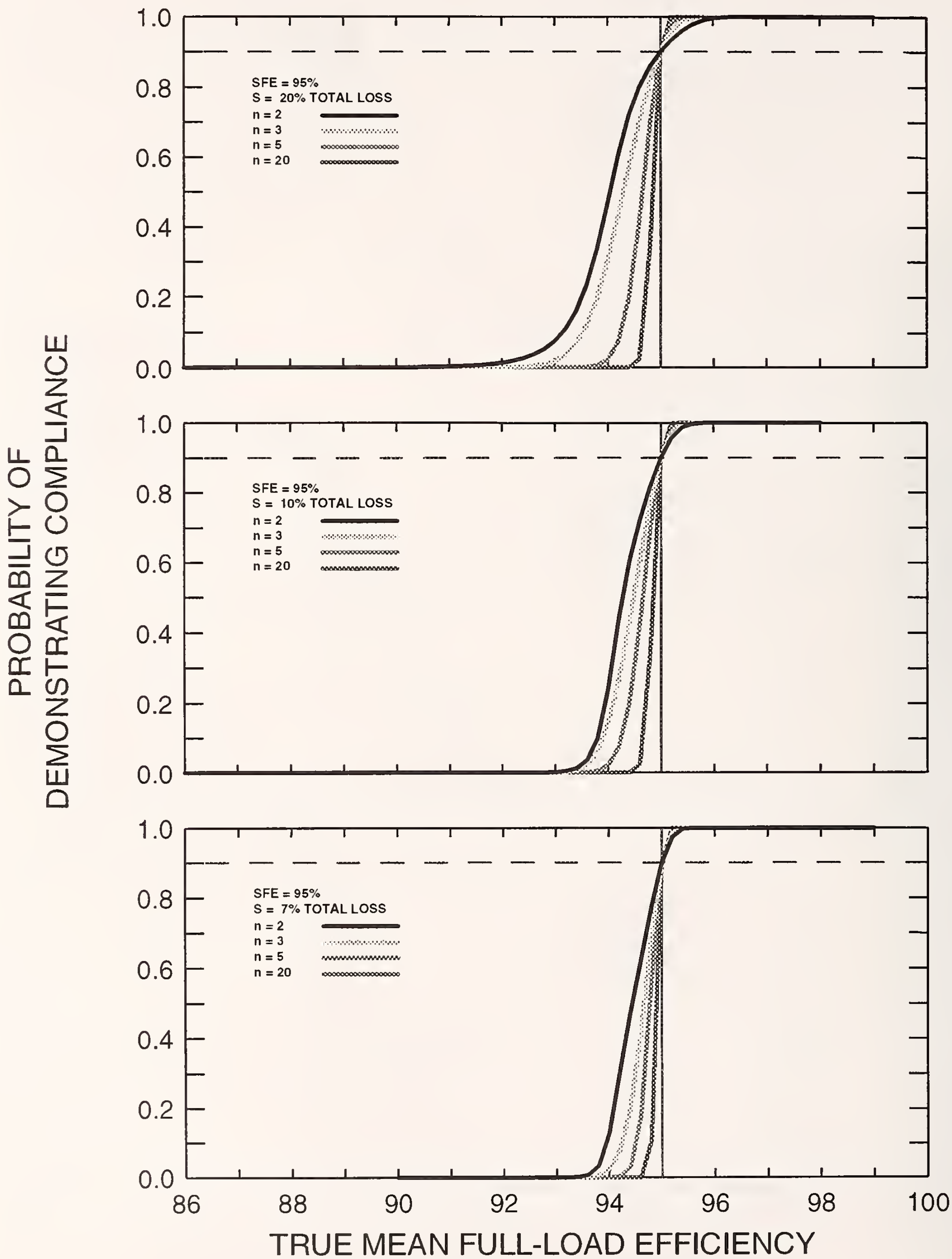

Figure 2. The probability of demonstrating compliance for sample size, n, and standard deviation, S. The statutory full-load efficiency, SFE, is 95 percent. The motor efficiencies are assumed to be normally distributed with the standard deviation expressed as a percentage of the total losses, i.e., $(100 - \mu)$, where μ is the true mean full-load efficiency.

Appendix A

Sampling Plan for Enforcement Testing

The Sampling Plan for enforcement as is published in the *Federal Register* [1] follows:

Step 1. The first sample size (n_1) must be five or more units.

Step 2. Compute the mean ($\bar{X}_1$) of the measured energy performance of the n_1 units in the first sample as follows:

$$\bar{X}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i, \tag{1}$$

where X_i is the measured full-load efficiency of unit i.

Step 3. Compute the sample standard deviation (S_1) of the measured full-load efficiency of the n_1 units in the first sample as follows:

$$S_1 = \sqrt{\frac{\sum_{i=1}^{n_1}(X_i - \bar{X}_1)^2}{n_1 - 1}}. \tag{2}$$

Step 4. Compute the standard error ($SE(\bar{X}_1)$) of the mean full-load efficiency of the first sample as follows:

$$SE(\bar{X}_1) = \frac{S_1}{\sqrt{n_1}}. \tag{3}$$

Step 5. Compute the lower control limit (LCL_1) for the mean of the first sample using the applicable statutory full-load efficiency (SFE) as the desired mean as follows:

$$LCL_1 = SFE - tSE(\bar{X}_1). \tag{4}$$

Here t is the 10th percentile of a t-distribution for a sample size of n_1 and yields a 90 percent confidence level for a one-tailed t-test.

Step 6. Compare the mean of the first sample ($\bar{X}_1$) with the lower control limit (LCL_1) to determine one of the following:

(i) If the mean of the first sample is below the lower control limit, then the basic model is in noncompliance and testing is at an end.

(ii) If the mean is equal to or greater than the lower control limit, no final determination of compliance or noncompliance can be made; proceed to Step 7.

Step 7. Determine the recommended sample size (n) as follows:

$$n = \left[\frac{tS_1(120 - 0.2SFE)}{SFE(20 - 0.2SFE)}\right]^2 \tag{5}$$

where S_1 and t have the values used in Steps 4 and 5, respectively. The factor

$$\frac{120 - 0.2SFE}{SFE(20 - 0.2SFE)}$$

is based on a 20 percent tolerance in the total power loss at full load and fixed output power.

Given the value of n, determine one of the following:

(i) If the value of n is less than or equal to n_1 and if the mean energy efficiency of the first sample ($\bar{X}_1$) is equal to or greater than the lower control limit (LCL_1), the basic model is in compliance and testing is at an end.

(ii) If the value of n is greater than n_1, the basic model is in noncompliance. The size of a second sample n_2 is determined to be the smallest integer equal to or greater than the difference $n - n_1$. If the value of n_2 so calculated is greater than $20 - n_1$, set n_2 equal to $20 - n_1$.

Step 8. Compute the combined mean ($\bar{X}_2$) of the measured energy performance of the n_1 and n_2 units of the combined first and second samples as follows:

$$\bar{X}_2 = \frac{1}{n_1 + n_2} \sum_{i=1}^{n_1+n_2} X_i. \tag{6}$$

Step 9. Compute the standard error ($SE(\bar{X}_2)$) of the mean full-load efficiency of the n_1 and n_2 units in the combined first and second samples as follows:

$$SE(\bar{X}_2) = \frac{S_1}{\sqrt{n_1 + n_2}}. \tag{7}$$

(Note that S_1 is the value obtained above in Step 3.)

Step 10. Set the lower control limit (LCL_2) to,

$$LCL_2 = SFE - tSE(\bar{X}_2), \tag{8}$$

where t has the value obtained in Step 5, and compare the combined sample mean ($\bar{X}_2$) to the lower control limit (LCL_2) to find one of the following:

(i) If the mean of the combined sample ($\bar{X}_2$) is less than the lower control limit (LCL_2), the basic model is in noncompliance and testing is at an end.

(ii) If the mean of the combined sample ($\bar{X}_2$) is equal to or greater than the lower control limit (LCL_2), the basic model is in compliance and testing is at an end.

MANUFACTURER-OPTION TESTING

If a determination of non-compliance is made in Steps 6, 7 or 11, above, the manufacturer may request that additional testing be conducted, in accordance with the following procedures.

Step A. The manufacturer requests that an additional number, n_3, of units be tested, with n_3 chosen such that $n_1 + n_2 + n_3$ does not exceed 20.

Step B. Compute the mean full-load efficiency, standard error, and lower control limit of the new combined sample in accordance with the procedures prescribed in Steps 8, 9, and 10, above.

Step C. Compare the mean performance of the new combined sample to the lower control limit (LCL_2) to determine one of the following:

(a) If the new combined sample mean is equal to or greater than the lower control limit, the basic model is in compliance and testing is at an end.

(b) If the new combined sample mean is less than the lower control limit and the value of $n_1 + n_2 + n_3$ is less than 20, the manufacturer may request that additional units be tested. The total of all units tested may not exceed 20. Steps A, B, and C are then repeated.

(c) Otherwise, the basic model is determined to be in noncompliance.

Appendix B
The probability of compliance

The Algorithm

If one makes the model assumptions that the population of motor efficiencies is normally distributed with mean μ and standard deviation σ, then the probability of compliance can be calculated using straightforward numerical integration.

In the following expressions, $n_1 \geq 5$ denotes the minimum sample size in the sampling plan, and $n_2 = 20$ is the maximum sample size 5 and 20, respectively). In order to simplify the equations, $n_1 - 1$ is represented by ν, and t is the 95th percentile of the t distribution for a sample size of n_1, which appears in the sampling plan.

The probability of compliance is

$$p = 1 - \frac{\left(\nu/\sigma^2\right)^{\nu/2}}{\Gamma_{\nu/2} 2^{\nu/2-1}} \sum_{i=n_1}^{n_2} \int_{\kappa_i}^{\kappa_i+1} \Phi \left[\frac{(\text{SFE} - \mu)\sqrt{i} - tx}{\sigma} \right] x^{\nu-1} e^{-\nu x/2\sigma^2} dx.,$$

where the limits of integration are $\kappa_{n_1} = 0$,

$$\kappa_i = \frac{\text{SFE}(20 - .2\text{SFE})}{(120 - .2\text{SFE})t} \sqrt{i-1} \text{ for } i = n_1 + 1, \ldots, n_2,$$

and $\kappa_{n_2+1} = \infty$.

The function $\Phi(u)$ is defined to be the probability that a normally distributed random variable is greater than u, that is

$$\Phi(u) = \int_{-\infty}^{u} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx.$$

There are many public-domain routines available for numerically calculating this integral efficiently and accurately. The complete gamma function with parameter $\nu/2$, $\Gamma_{\nu/2}$, can also be calculated without writing special software.

The FORTRAN Code

A partial listing of the computer source code used for the model calculations.

```
      implicit double precision (a-h, o-z)
C
C     Test problem for probability of compliance
C     calculation. The correct output is:
C
C   Prob. of compliance =   0.4163048163619565
C
      dimension x(500), w(500)
      nquad = 500
      n1    = 5
      xmu    = 88
      sfe   = 90
      sig   = dfloat(100-xmu)/3
      x(1)  = -1
      call comply (n1, xmu, sig, sfe, prob, x, w, nquad)
      write (*,*) ' Prob. of compliance = ',prob
      stop
C
      end


      SUBROUTINE COMPLY
     $  (n1, xmu, sig, sfe, prob, xvec, wvec, nquad)
C***BEGIN PROLOGUE  COMPLY
C***DATE WRITTEN   950530   (YYMMDD)
C***REVISION DATE  000000   (YYMMDD)
C***CATEGORY NO.
C***KEYWORDS
C***AUTHOR  VANGEL, M.,
C***(NATIONAL INSTITUTE OF STANDARD AND TECHNOLOGY)
C***PURPOSE Calculates the probability of compliance for motors tested
     according to the proposed test plan.
C***DESCRIPTION
C
C     Abstract
C
C     This subroutine calculates the probability of compliance using
C     Gauss-Legendre quadrature.  The first time you call the
C     subroutine, set xvec(1) to some negative value. The quadrature
C     abscissas will be calculated and returned in 'xvec', and the
C     quadrature weights will be returned in 'wvec'.  In subsequent
C     calls, these weights and abscissas will not be recalculated unless
C     x(1) is set to a value LE 0.
C
C     Description of Parameters
C
C     --Input--
C      n1      - Minimum no. of motors tested (integer, input)
C      xmu   - Mean efficiency of population (in %) (double precision, input)
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```
C      sig      - Standard deviation of population (in %) (double precision, input)
C      sfe      - Statutory full-load efficiency (in %) (double precision, input)
C      prob    - Probability of compliance (double precision, output)
C      xvec   - Vector of length 'nquad' of abscissas (double precision, input/output)
C      wvec  - Vector of length 'nquad' of weights (double precision, input/output)
C      nquad  - Number of Gauss-Legendre quadrature points (integer, input)
C                   (nquad should be at least 50. To be safe, set nquad = 100
C                    or even nquad = 500.)
C
C     --Output--
C      (NONE)
C
C***REFERENCES  NONE
C***ROUTINES CALLED GAULEG, TPPF, NORMP
C***END PROLOGUE COMPLY
      implicit double precision (a-h, o-z)
      real preal, t
      dimension xvec(1), wvec(1), crit(21)
      data zero, one/0.d0, 1.d0/
C
C     -- If xvec(1) LE 0, then calculate the quadrature abscissas and weights, otherwise
C         assume that the these values have already been calculated.
C
C***FIRST EXECUTABLE STATEMENT COMPLY
      if (xvec(1) .le. 0) then
         call gauleg(zero, one, xvec, wvec, nquad)
      end if
C
C     -- Get the appropriate t-quartile
      preal = .90E0
      call tppf (preal, n1-1, t)
C
C     -- Calculate the points at which n changes
      con     = sfe*(20-.2d0*sfe)/((120-.2d0*sfe)*t)
      crit(n1-1) = 0
      do 10 i=n1, 19
         crit(i) = sqrt(dfloat(i)) *con
 10   CONTINUE
C
C     -- Pick a big number to 'approximate infinity' for upper limit of final integral.
      crit(20) = 1000 *sig/sqrt(dfloat(n1))
C
C     -- Do the integrals
      prob = 0
      do 20 i=n1-1, 19
         dlt = crit(i+1) -crit(i)
         d0  = crit(i)
         do 30 j=1, nquad
C
C     -- Weights and abscissas were calculated for integration on [0,1].  They must be
C         transformed for integration on [crit(i), crit(i+1)].
            x = d0 +dlt*xvec(j)
            w = dlt*wvec(j)
```

```
C
C    -- Argument for standard normal distribution
            arg = (sqrt(dfloat(i+1))*(sfe-xmu) -t*x)/sig
C
C     -- Upper tail probability for normal is q
            call normp (arg, p, q, pdf)
C
C     -- Here the argument of the integrand is calculated.
            arg = log(q) +dfloat(n1-1)/2 *
     $            log((n1-1)/sig**2) +
     $            (n1-2)*log(x) -(n1-1)*x**2/(2*sig**2) -
     $            (dfloat(n1-1)/2-1)*log(dfloat(2)) -
     $            dlngam(dfloat(n1-1)/2)
            arg  = exp(arg)
C
C     -- The numerical integrations are performed by accumulating a weighted sum
C         of the values of the integrands.
            prob = prob +arg *w
 30     CONTINUE
 20   CONTINUE
C
      RETURN
C
      END
```

Appendix C
An abridged t-table

The t values covering the range of sample sizes and confidences for the Sampling Plan are presented below.

Table C1. An abridged table of t percentiles

Sample	Confidence		
Size n	90%	95%	99%
2	3.078	6.314	31.82
3	1.886	2.920	6.965
4	1.638	2.353	4.541
5	1.533	2.132	3.747
6	1.476	2.015	3.365
7	1.440	1.943	3.143
8	1.415	1.895	2.998
9	1.397	1.860	2.896
10	1.383	1.833	2.821
11	1.372	1.812	2.764
12	1.363	1.796	2.718
13	1.356	1.782	2.681
14	1.350	1.771	2.650
15	1.345	1.761	2.624
16	1.341	1.753	2.602
17	1.337	1.746	2.583
18	1.333	1.740	2.567
19	1.330	1.734	2.552
20	1.328	1.729	2.539

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